PRINCETON UNIVERSITY

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E-mail: jwai@princeton.edu

Feedforward + feedback shape control design on NSTXU

J. Wai[1], M.D. Boyer[2], W. Wehner[3], A.S. Welander[3], E. Kolemen[1,2]

[1]*Princeton University, Princeton NJ, USA*
[2]*Princeton Plasma Physics Laboratory, Princeton NJ, USA*
[3]*Genearal Atomics, San Diego CA, USA*

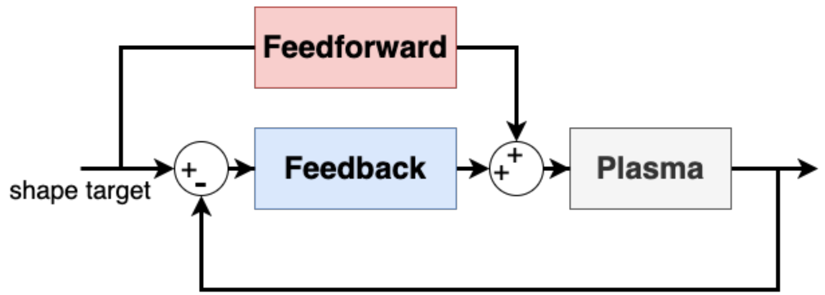
PPPL

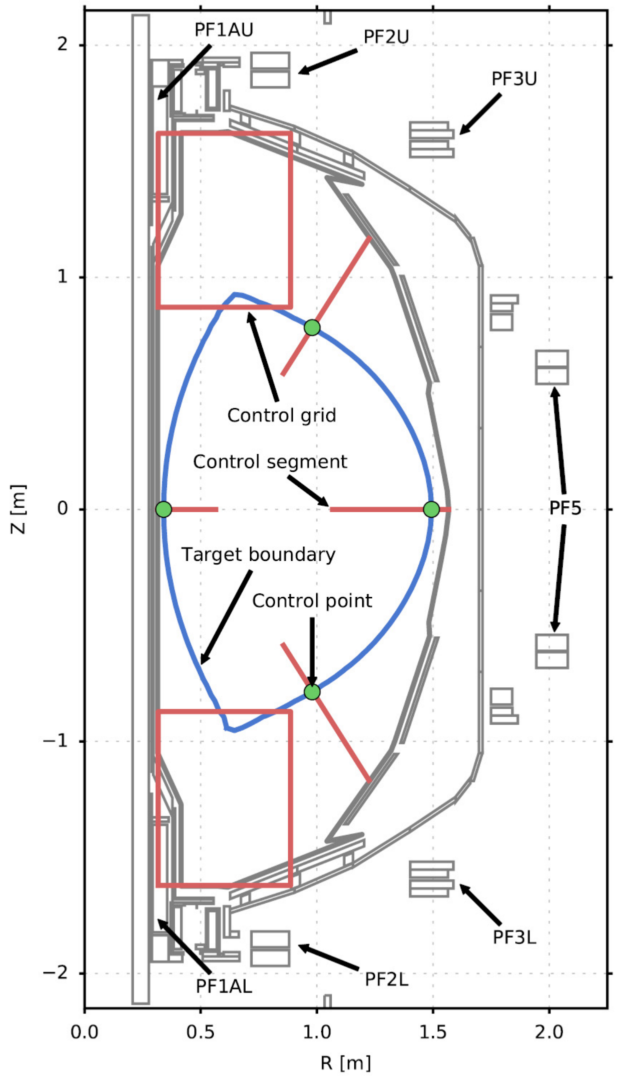
NSTX-U

Princeton Plasma Control
control.princeton.edu

Motivation

- Goal: improve NSTX-U shape controller
- Previous controller experienced difficulties
  - oscillations, sensitivity to gains, loss of control
- Target upgrades:
  - Add feedforward capabilities and feedforward design tool
  - Improve integration with Ip-controller and vertical stability controller





[Boyer, 2018]

Feedforward trajectory design

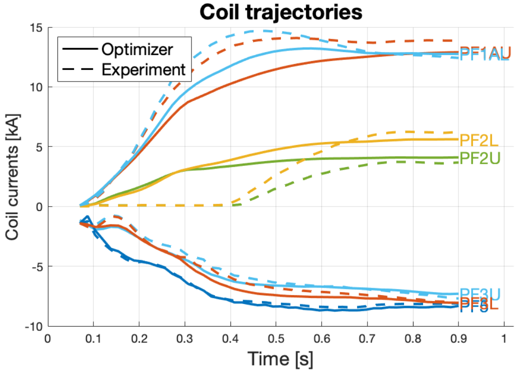
Feedforward (FF) design tools maps target shapes to currents

- Method requires only target shapes and estimates of a few scalar plasma parameters
  - Inputs: target Ip, target shapes, estimated Te, estimated W_{th}, estimated li
  - Results are not too sensitive parameters

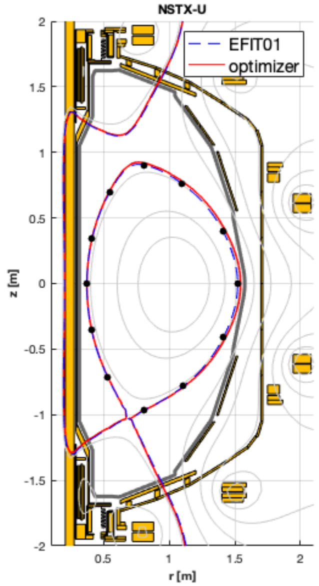
- Steps:
  - solve for equilibria at a few times
  - use coil/vessel/plasma dynamics to solve for ohmic and vessel currents
  - lock vessel currents and ohmic currents and repeat

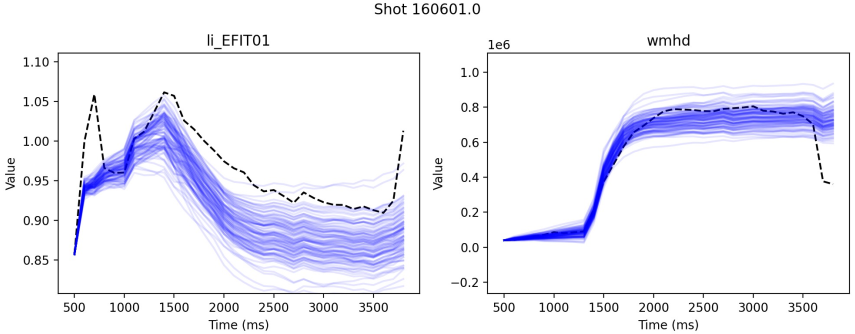
- Recurrent neural networks show promise in predicting these scalar parameters based on actuators *[I. Char, Carnegie Mellon University]*

$$M\dot{I} + RI = v$$



FF trajectories based on shot 204660





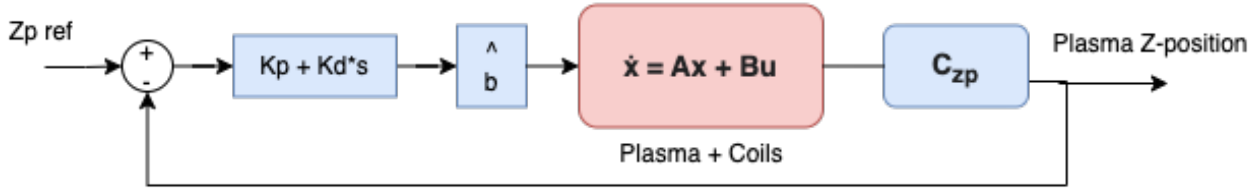
Parameter predictions trained on heating & current drive actuators

Vertical stability analysis

Circuit dynamics model gives insight into vertical stability

- Shape model is based on circuit equation, applied to toroidal elements in the tokamak

$$\left(M + \frac{\partial \psi_{pla}}{\partial I}\right)\dot{I} + RI = v \quad \longrightarrow \quad \begin{aligned} \dot{I} &= AI + Bv \\ A &= -(M+X)^{-1}R \\ B &= (M+X)^{-1} \end{aligned}$$



- Vertical instability is represented by a positive eigenvalue of A
- Analytic theory indicates proportional-derivative control is needed to stabilize system unless elongation is low *[Humphreys 1989, Lazarus 1990]*
- Theory also suggests presence of right-half-plane (RHP) transmission zeros

Zero: "values of s for which u and x are nonzero, but y is zero"

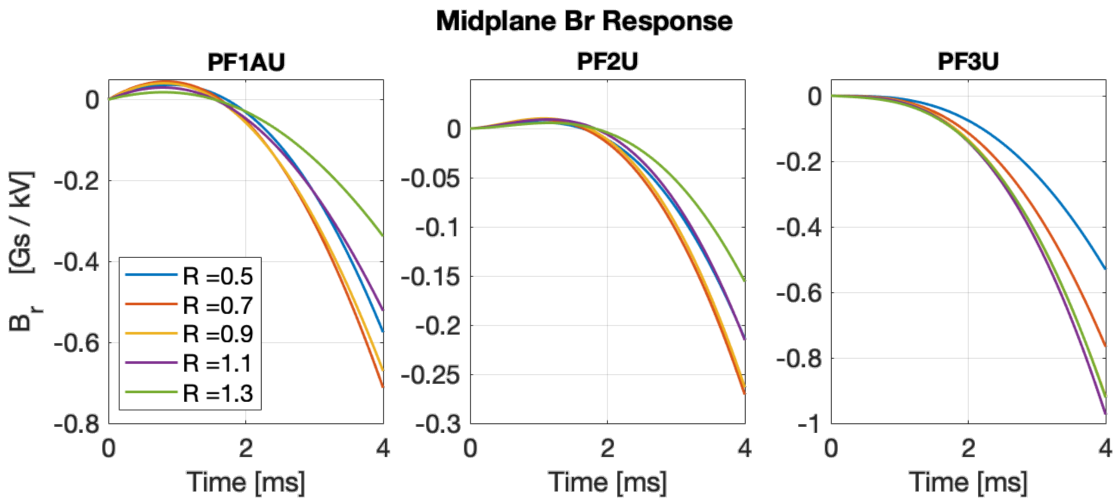
$$\begin{aligned} y(s) &= G(s)u(s) \\ &= \frac{n(s)}{d(s)}u(s) \end{aligned}$$

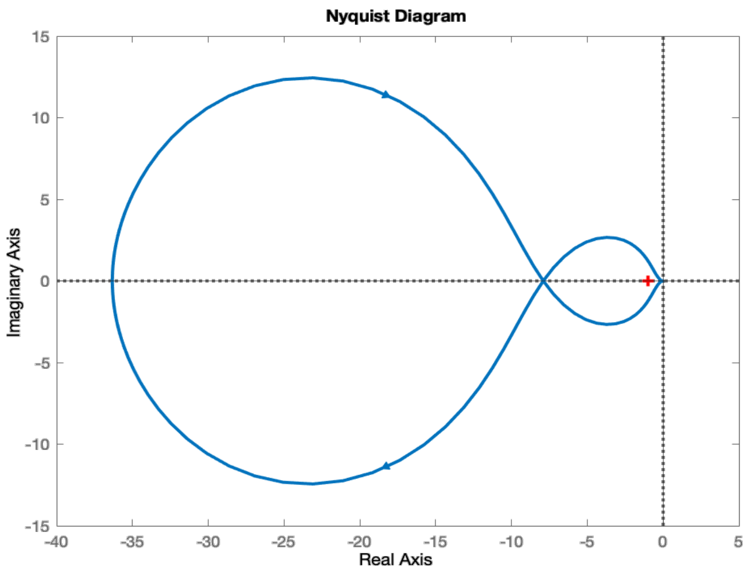
$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

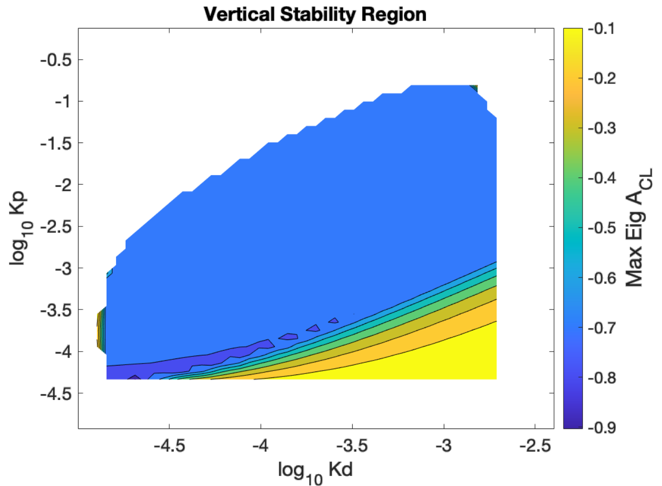
$$\begin{bmatrix} sI - A & B \\ C & D \end{bmatrix}$$ drops rank

Stabilizing region for vertical controller identified

- RHP zero in plasma position response is exactly the RHP zero in the vacuum field Br response *[Pesamosca 2021]*
  - On NSTX-U, the zero exists for PF1 and PF2 only due to vessel shielding
  - Fast timing (> 1kHz) suggests PF1 and PF2 are still fast enough to be used for vertical control
- Identified stable region for controller Kp, Kd values







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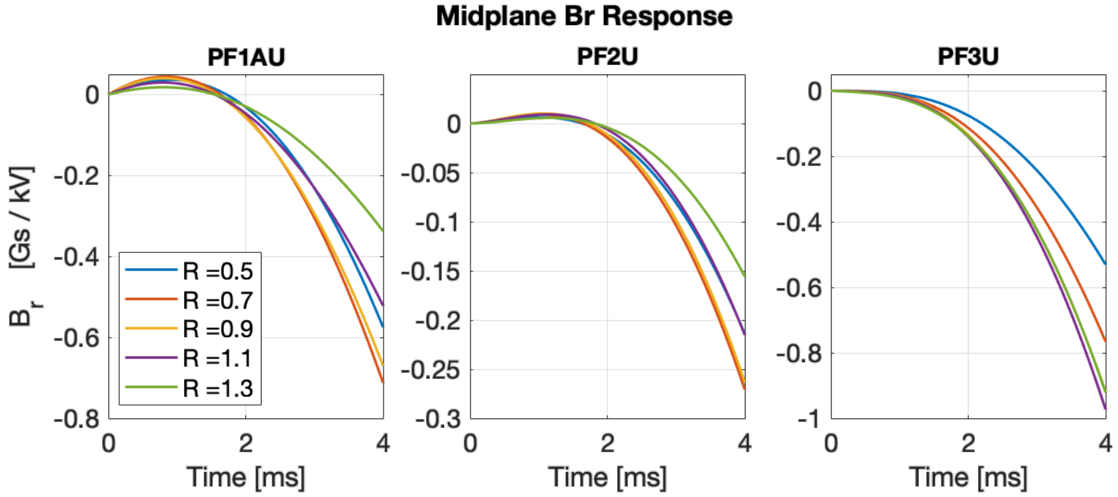

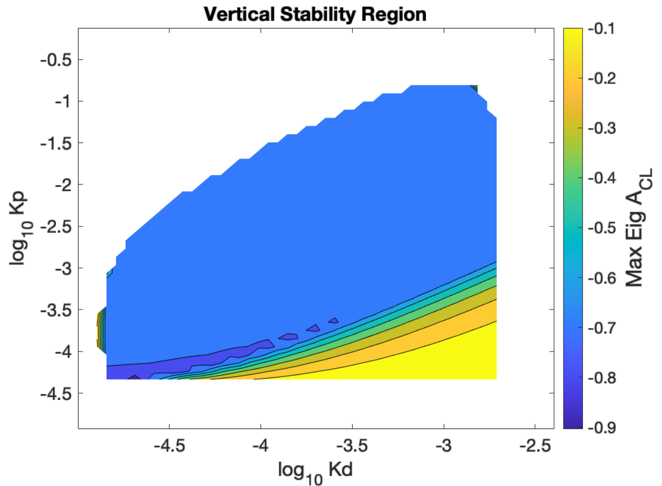

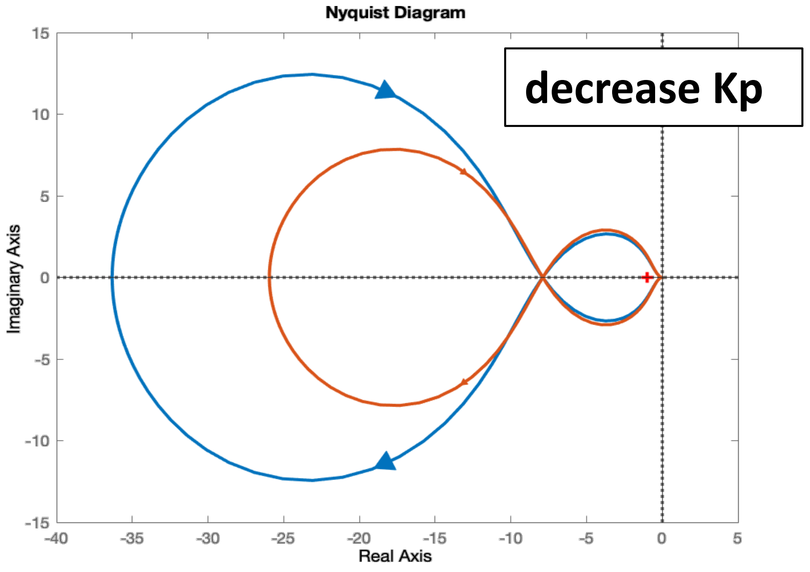

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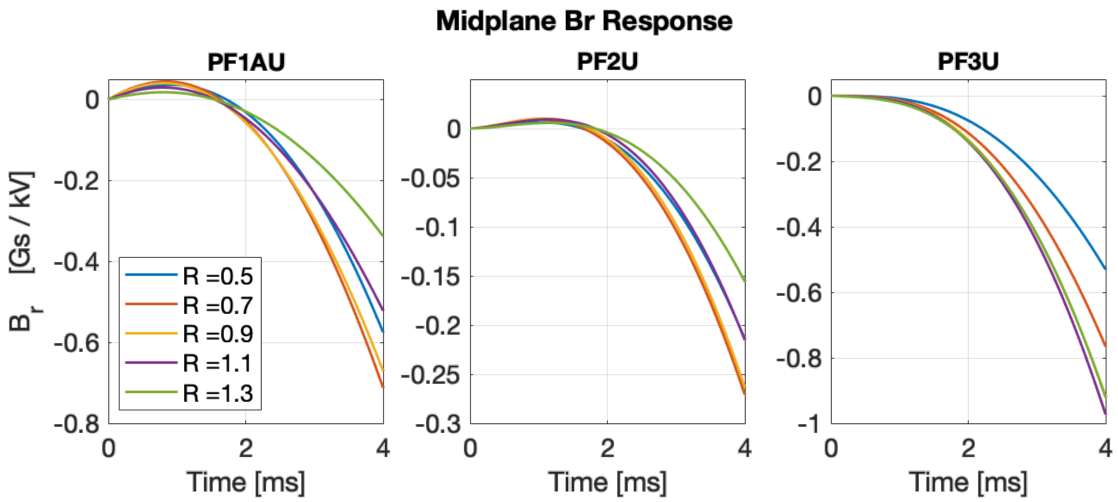
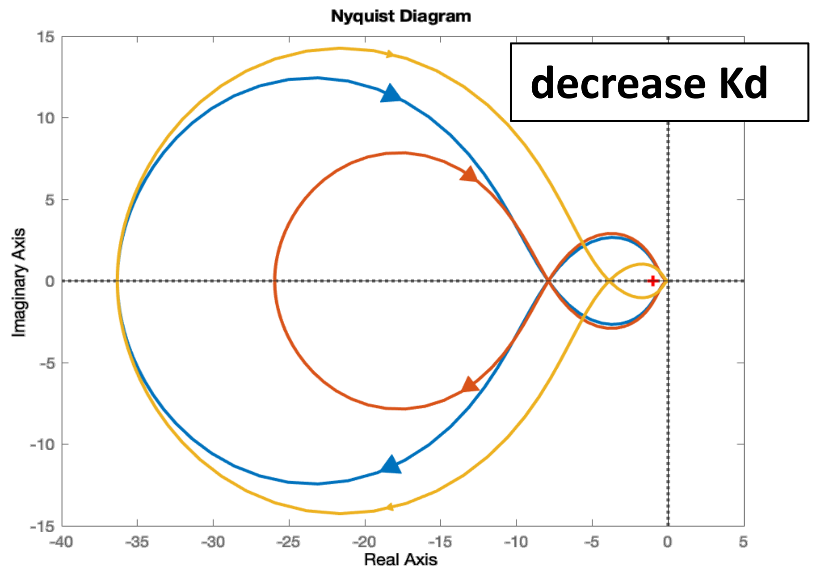
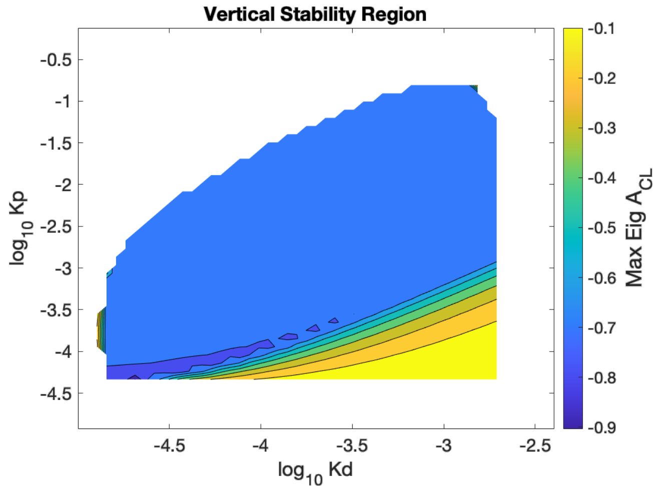


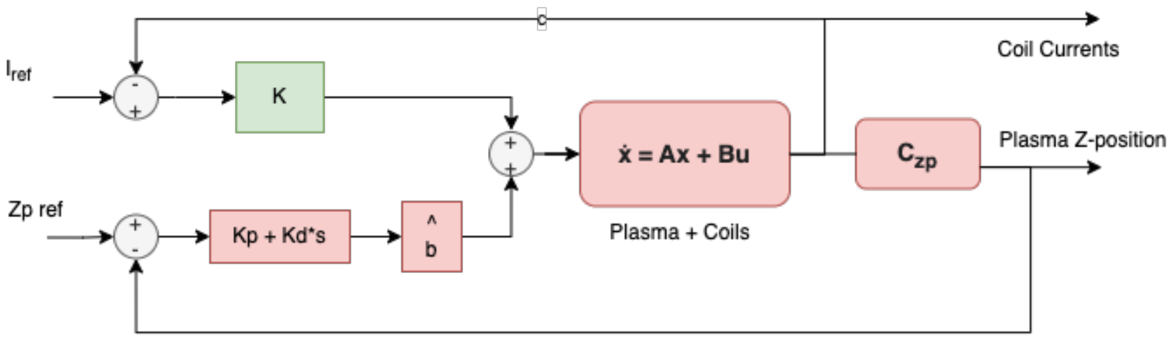




Shape and current-tracking

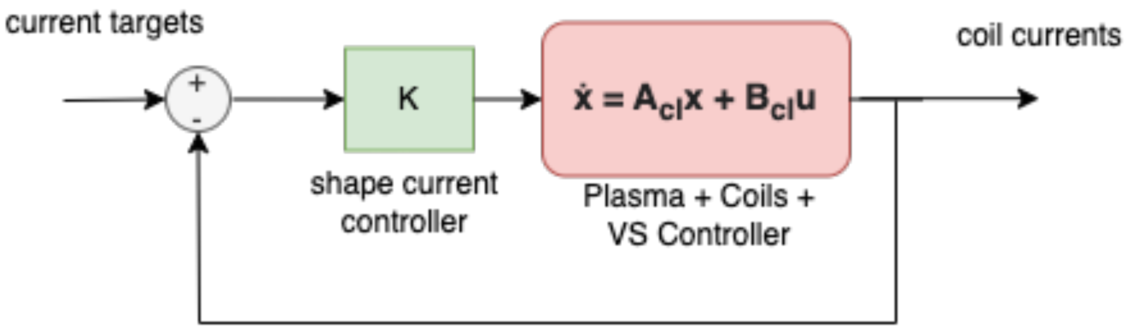
Vertical instability introduces a RHP zero to the PF current control loop

- "Closing the vertical loop" results in a RHP zero to the current/shape control loop
  - Fundamentally related to the vertical instability and has same timescale (10-200 Hz)
  - In general, only solution is to reduce controller agressiveness (bandwidth)



$$null\left(\begin{bmatrix} sI - A & B \\ C & D \end{bmatrix}\right)$$

$$\begin{bmatrix} sI_{n\times n} - A + (k_p + k_d s)B\hat{b}C_{zp} & B \\ [I_{m\times m} \;\; 0_{m\times(n-m)}] & 0_{m\times m} \end{bmatrix} \begin{bmatrix} x_0 \\ u_0 \end{bmatrix} = 0$$



Gives approximately:

$$(sI - A + v(s, x_0))x_0 = 0$$

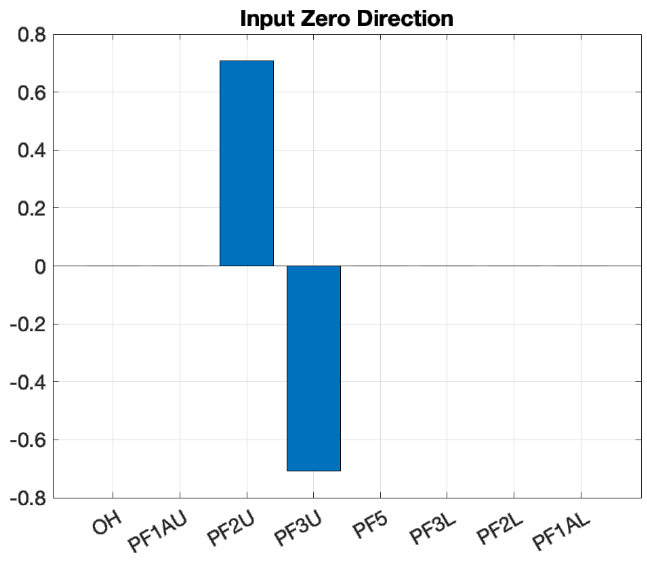
$$B[\alpha(s, x_0)\hat{b} + u_0] = 0$$

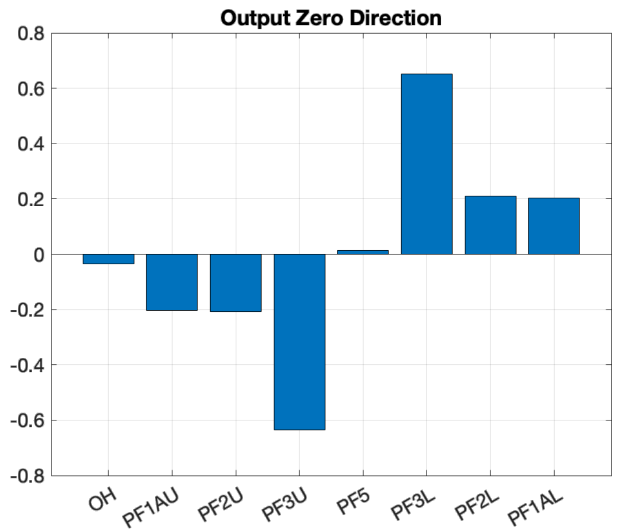
The zero is a perturbation to the solution of (sI-A)=0, the poles of A

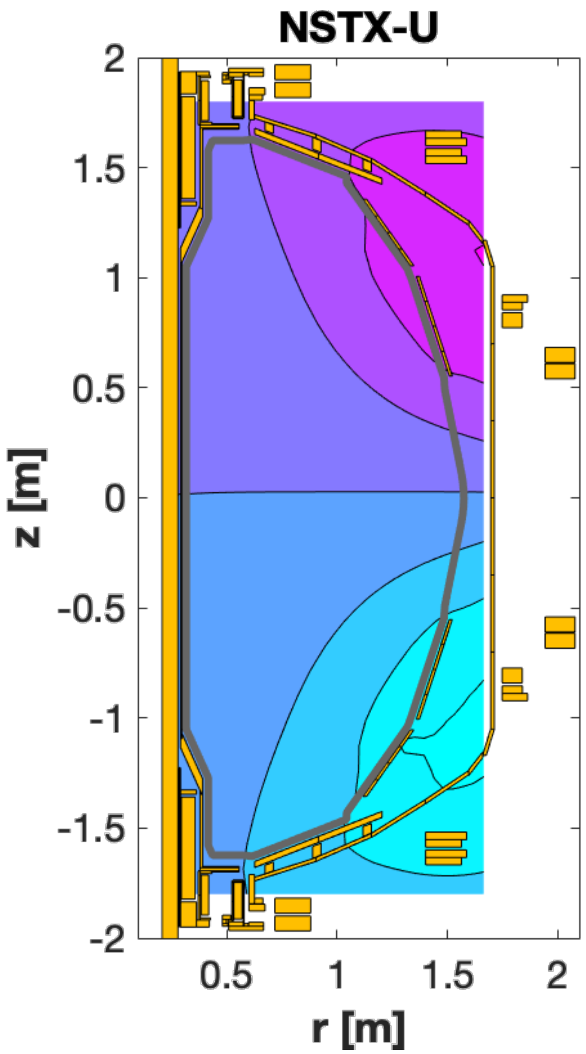
The input zero direction (u0) is ~ the vertical control input direction $\hat{b}$

Numerical calculation of RHP zero shows alignment with vertical instability

- Input direction u0 = null(G)
- Output direction y0 = null(G')
- Force actuation to be orthogonal to input zero direction, or force coil tracking errors to be orthogonal to output zero direction
- Misalignment between input and output zero directions indicates the VS controller would improve by adding PF1/PF2.







Full controller is based on current-following + shape error mapping

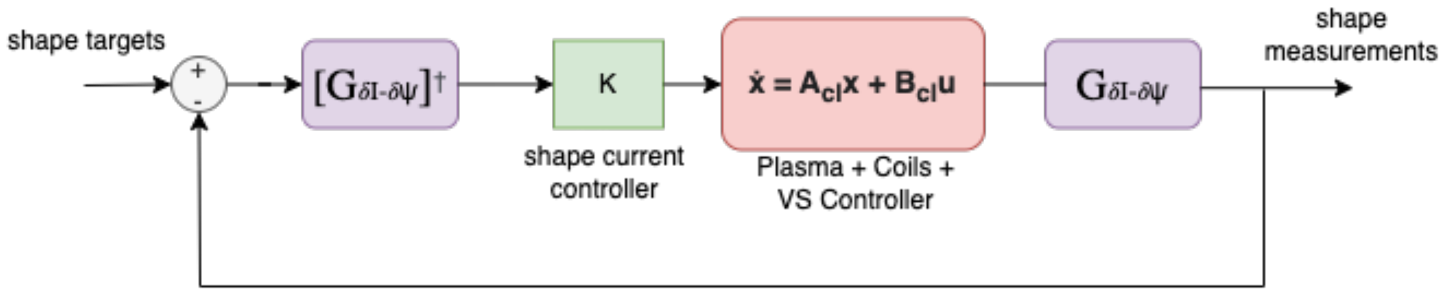
- Similar to the eXtreme Shape Controller at JET *[Ariola 2005]*
- Dynamic performance is mostly a function of the current controller
  - current control dynamics

$$\dot{x} = (A - BK)x$$

  - shape dynamics

$$\dot{x} = (A - BKG^{\dagger}G)x$$

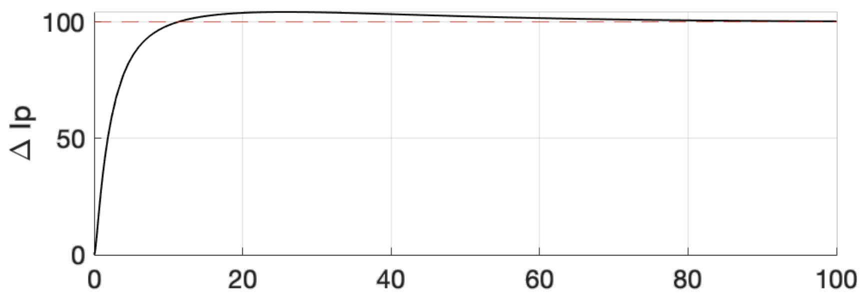
  - flexibility: design dynamic response independent of shape targets and shape scenario



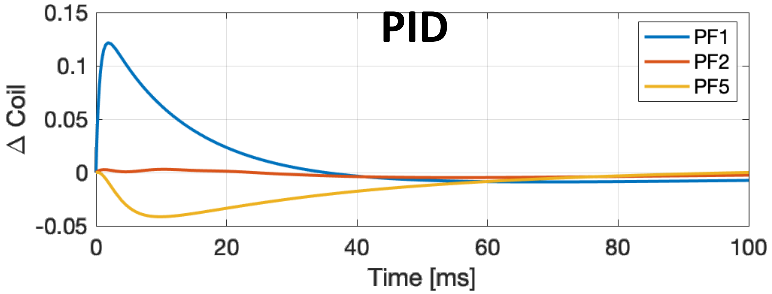
Surprisingly, PID current-tracking performs on-par with MIMO methods

- Highest coupling is between OH coil and PF1AU/PF1AL which are directly adjacent

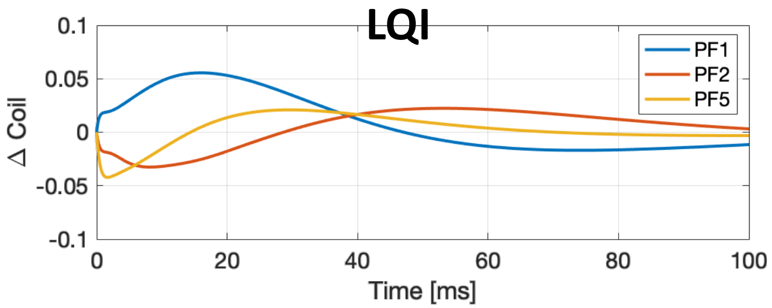
- Apply a step reference change in Ip
  - PID tracking rejects disturbance ~ 30ms
  - LQI and LQR can give some improvements/tradeoffs



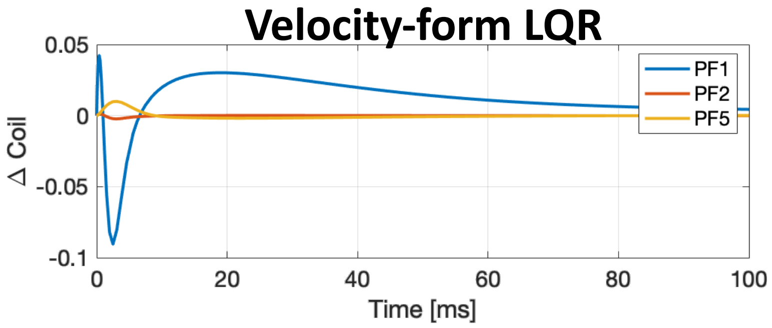
$$v = k_p x + k_i \int x dt + k_d \dot{x}$$



$$v = K_{LQI} \begin{bmatrix} x \\ \int edt \end{bmatrix}$$

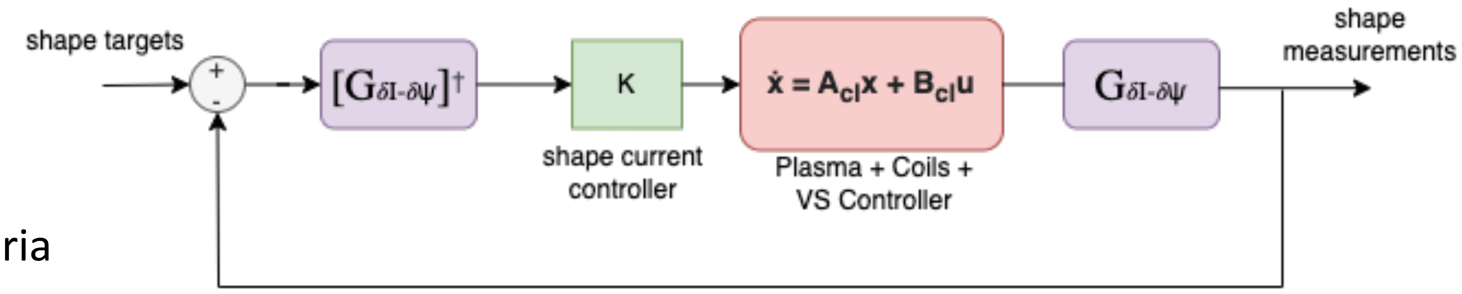


$$\dot{v} = K_{LQR} \begin{bmatrix} x \\ v \end{bmatrix}$$



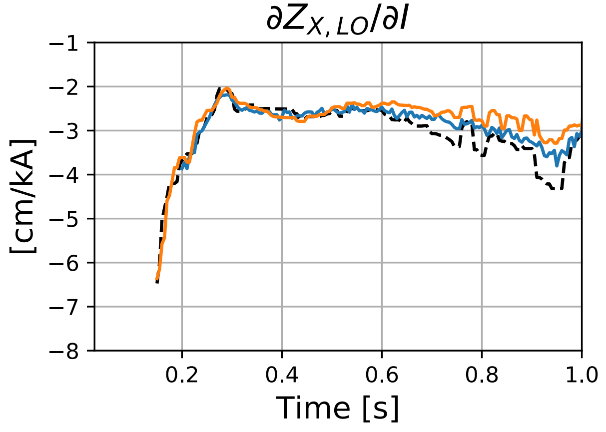
Map from shape errors to currents is the "plasma response"

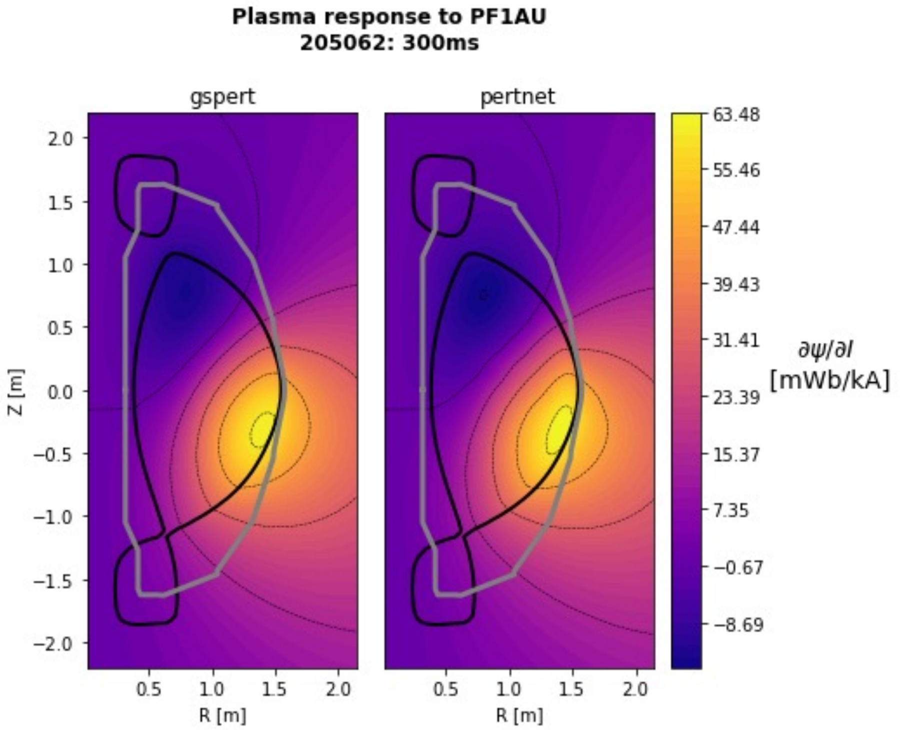
- This map is equilibrium dependent, linearization of the Grad-Shafranov equation
  - Precompute a-priori based on target equilibria
  - Simpler "rigid" model is real-time capable although not routinely used
  - Use plasma response neural network (Pertnet) *[Wai 2022]*



$$\delta\psi = G\delta I$$

$$\delta I = G^\dagger \delta\psi$$





Shape-to-current mapping can be used for constraints, including feedforward

- On JET XSC shape-to-current mapping is regularized using SVD *[Ariola 2005]*
  - Only retain the first few singular values

$$\delta I = G^{\dagger}\delta\psi$$

- Interpreting matrix inversion is intuitive for including feedforward, some types of constraints

$$\delta I = \text{argmin } J(\delta I) = ||\delta\psi - G\delta I||^2$$

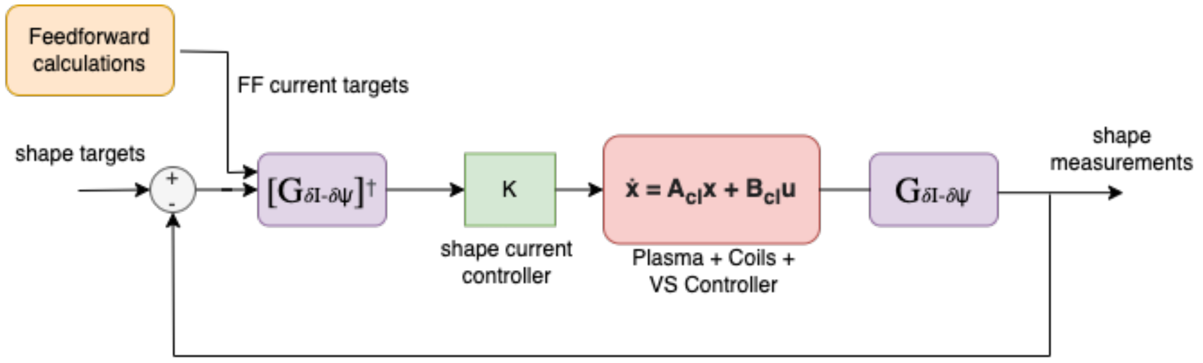
- Include weighting matrices, regularization, and constraints

min:

$$J = \delta I^T H \delta I + 2 f^T \delta I$$
$$H = G^T W_{\psi} G + W_I$$
$$f = G^T W_{\psi} \delta\psi$$

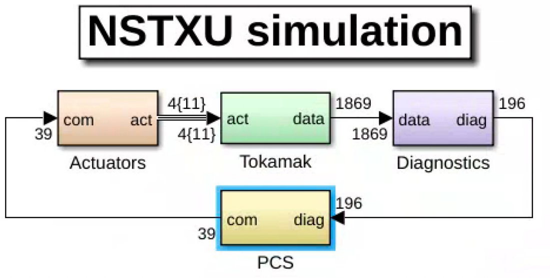
subject to:

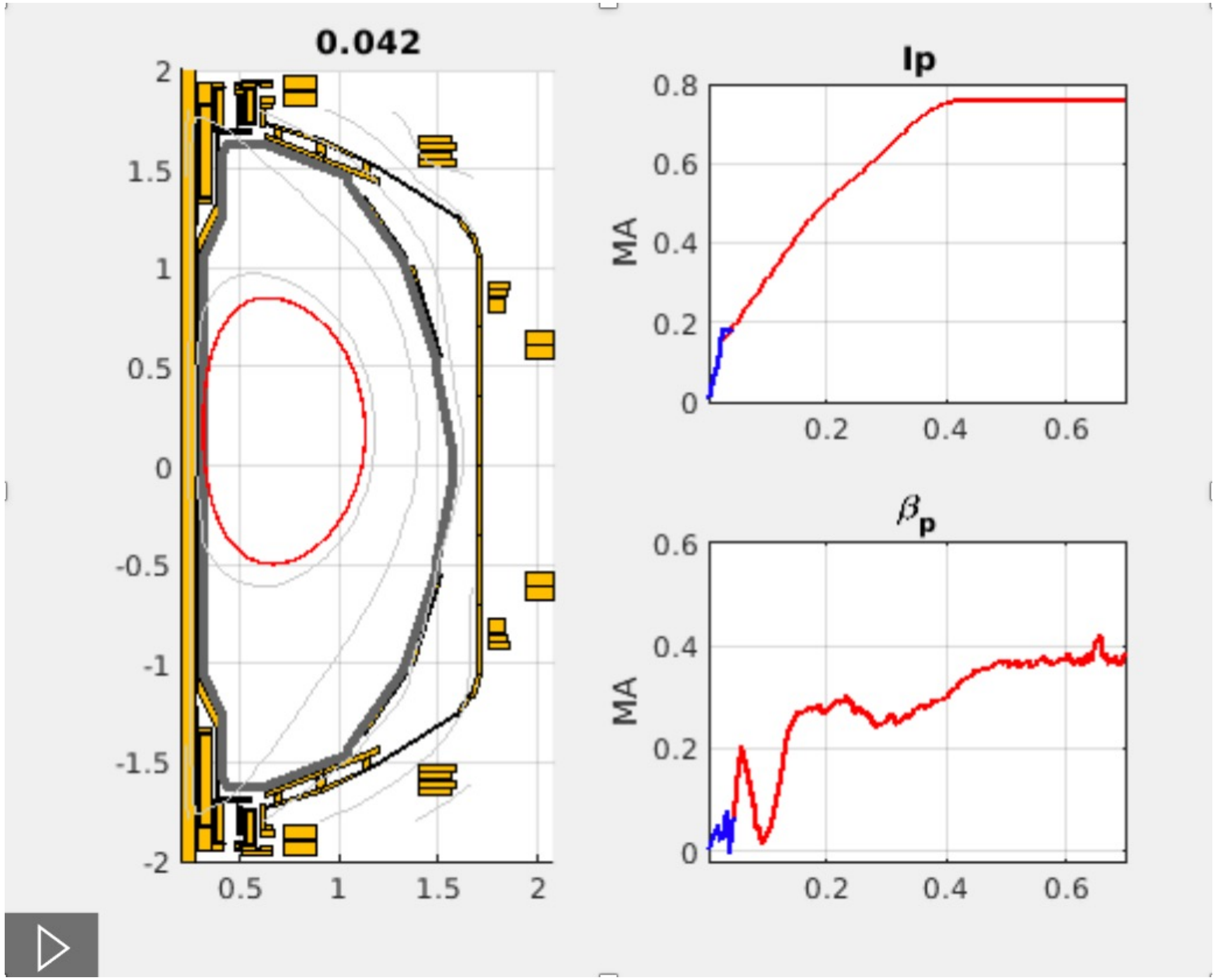
$$A\delta I < b$$



Nonlinear simulations performed using gsevolve [*Welander 2019*]

- Recreate shot using original PCS controller
  - experiment-level disturbances and noise
  - undesired USN-LSN bobble occurs while diverting
  - radial position oscillations
  - Ip oscillations (higher than actual experiment)





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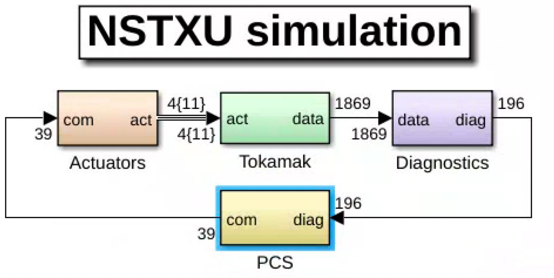

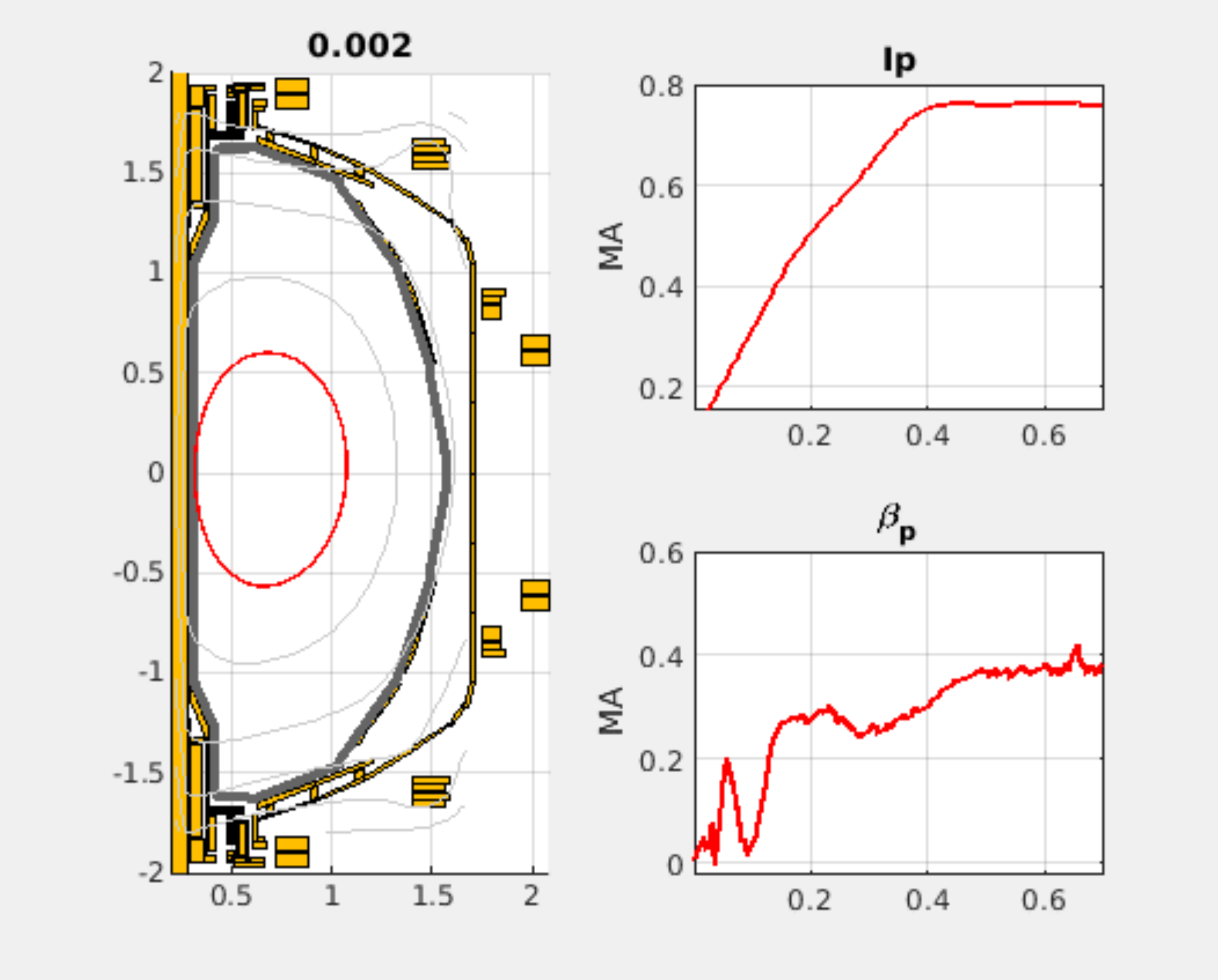

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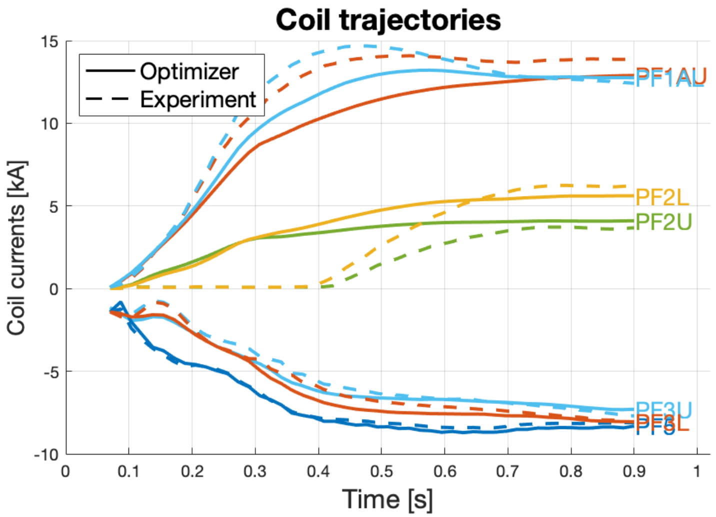
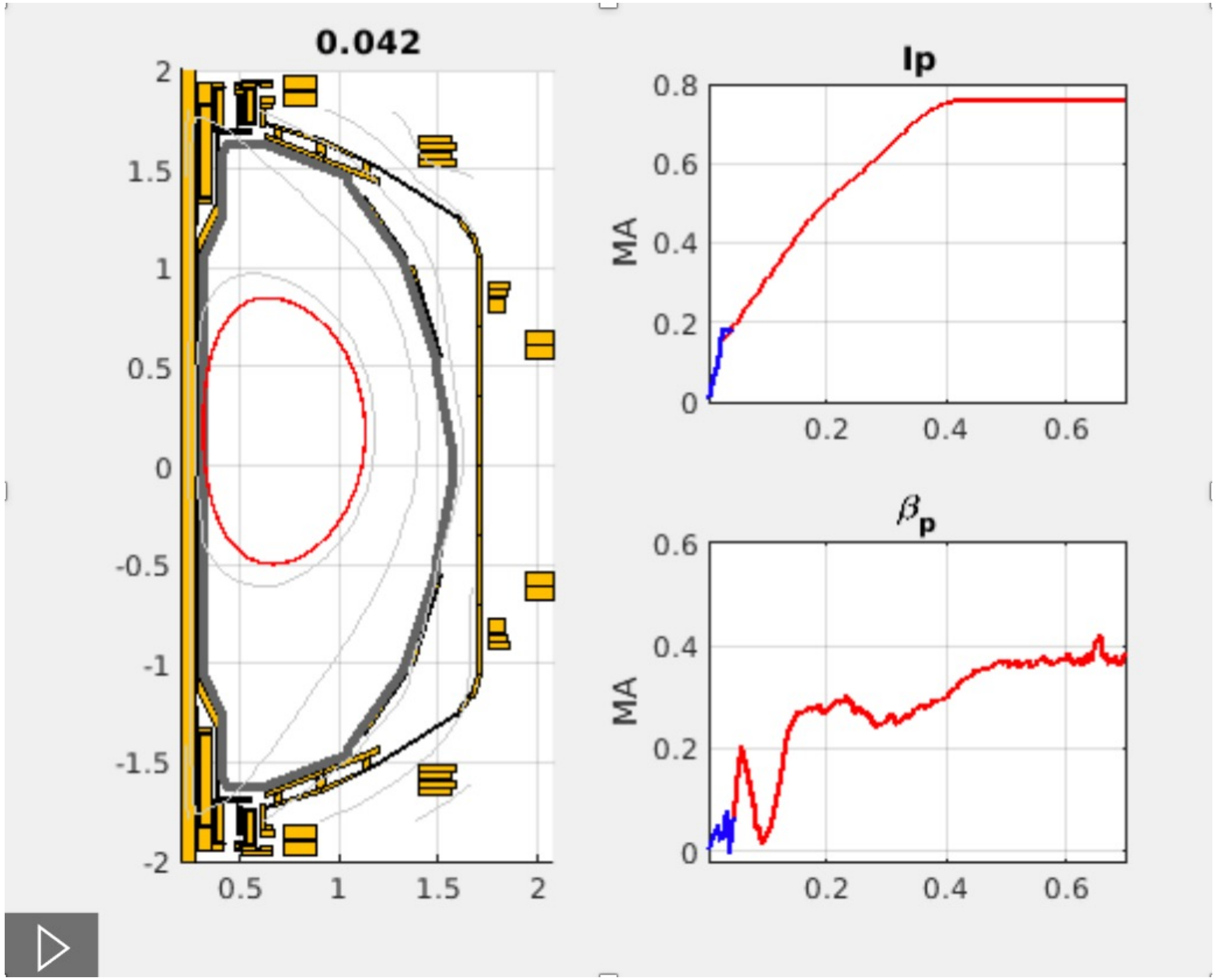
- Use feedforward method to design coil current trajectories
  - feedforward reduces PF1 currents while diverting, removes USN/LSN switching





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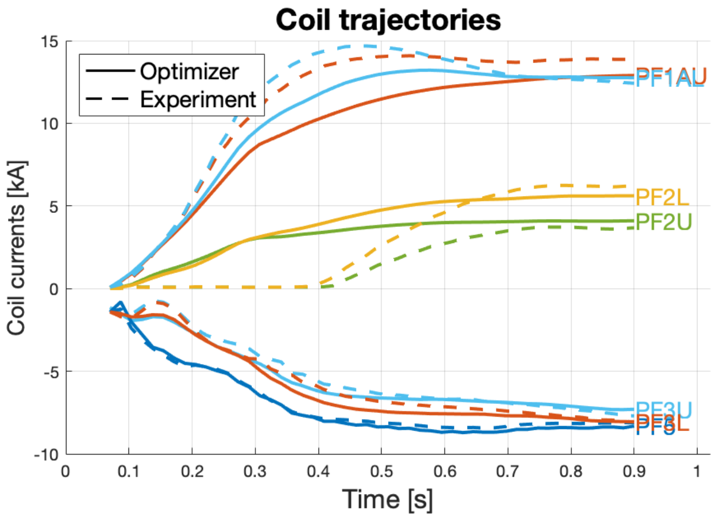

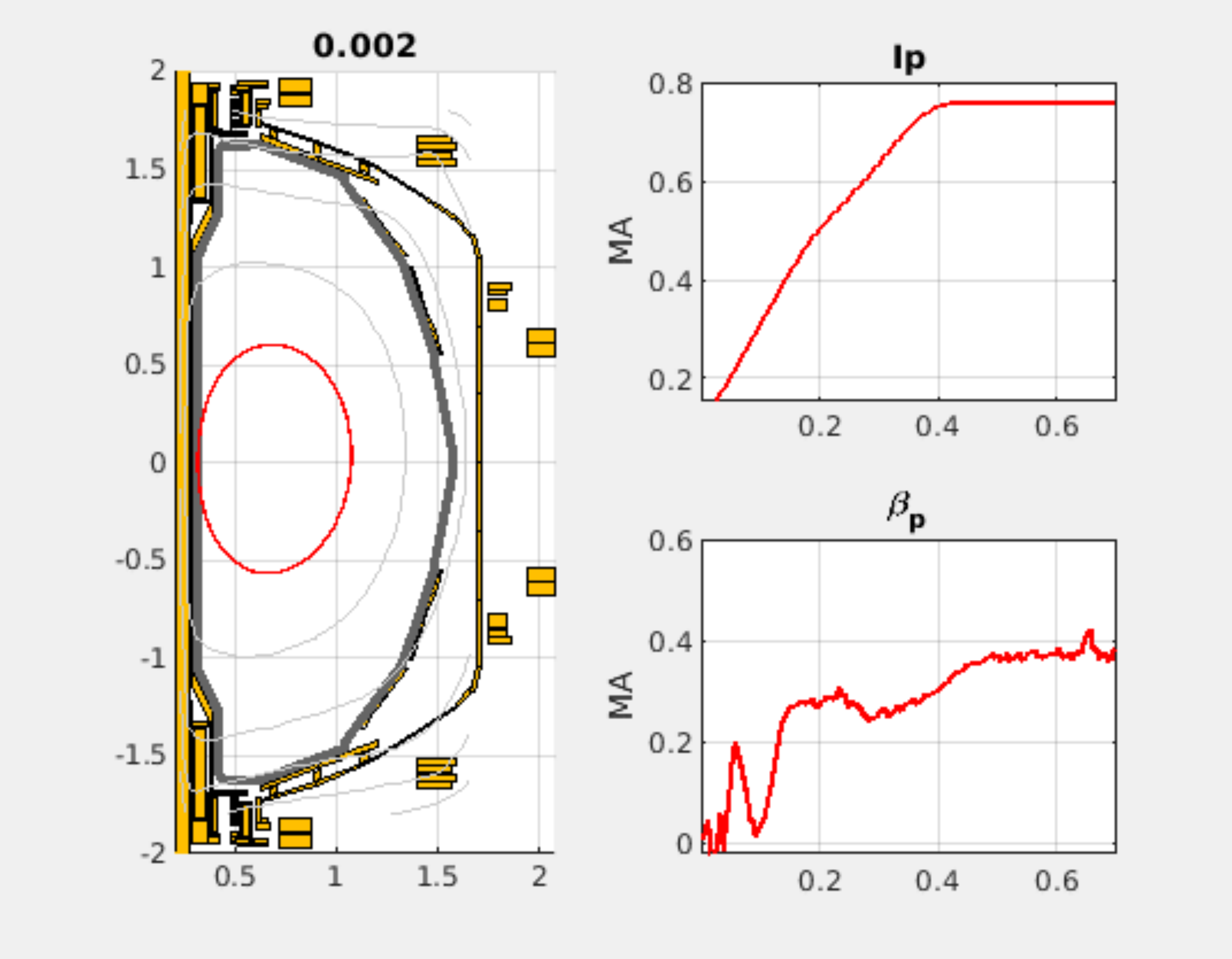

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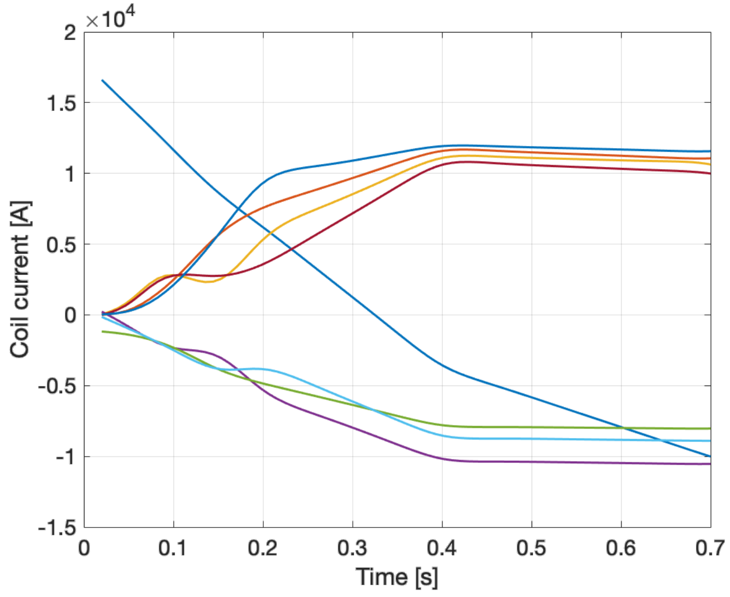

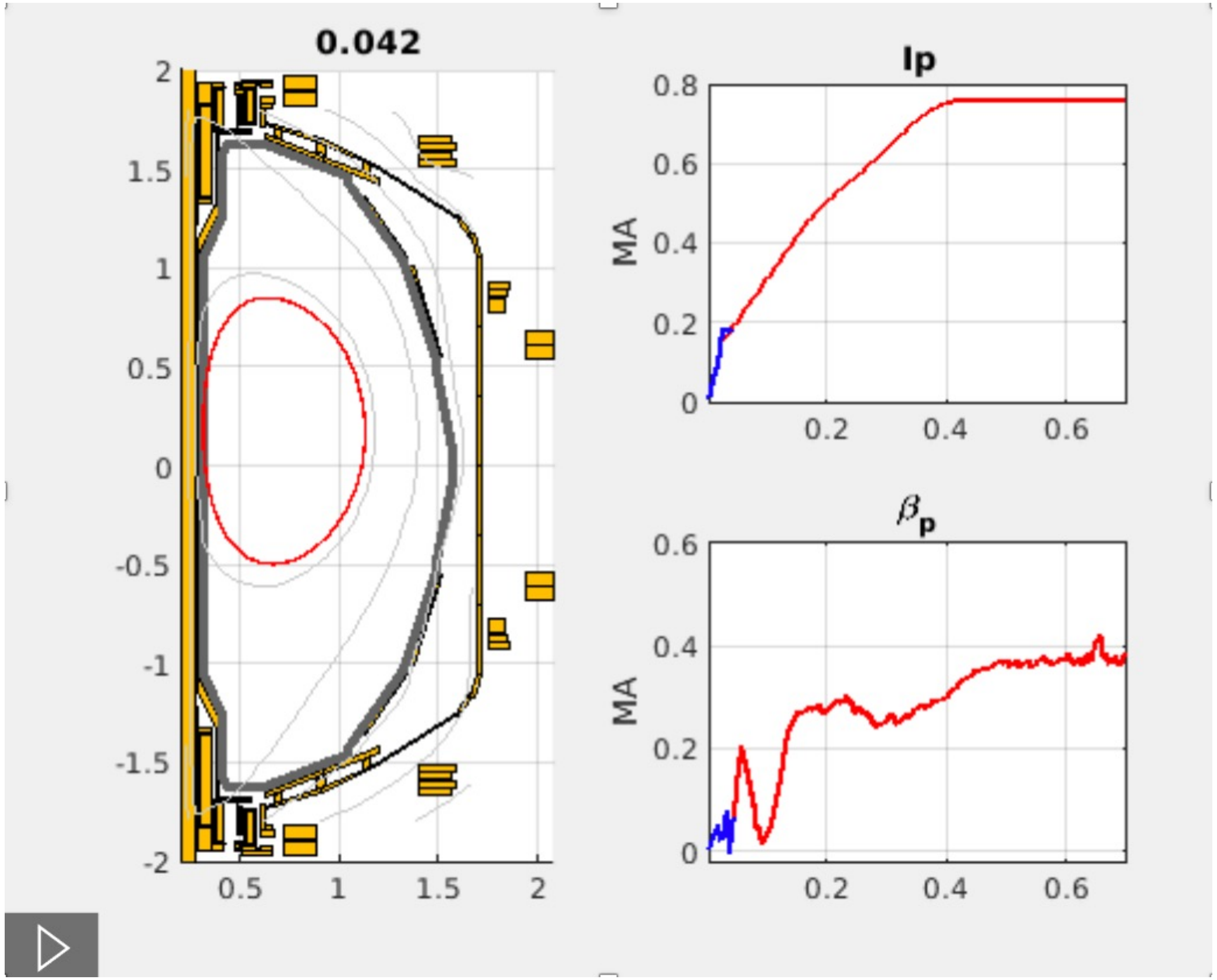

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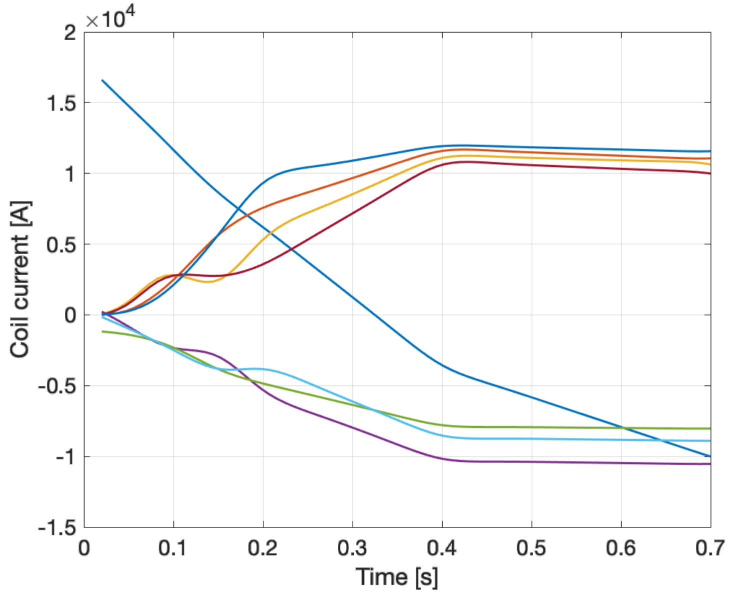

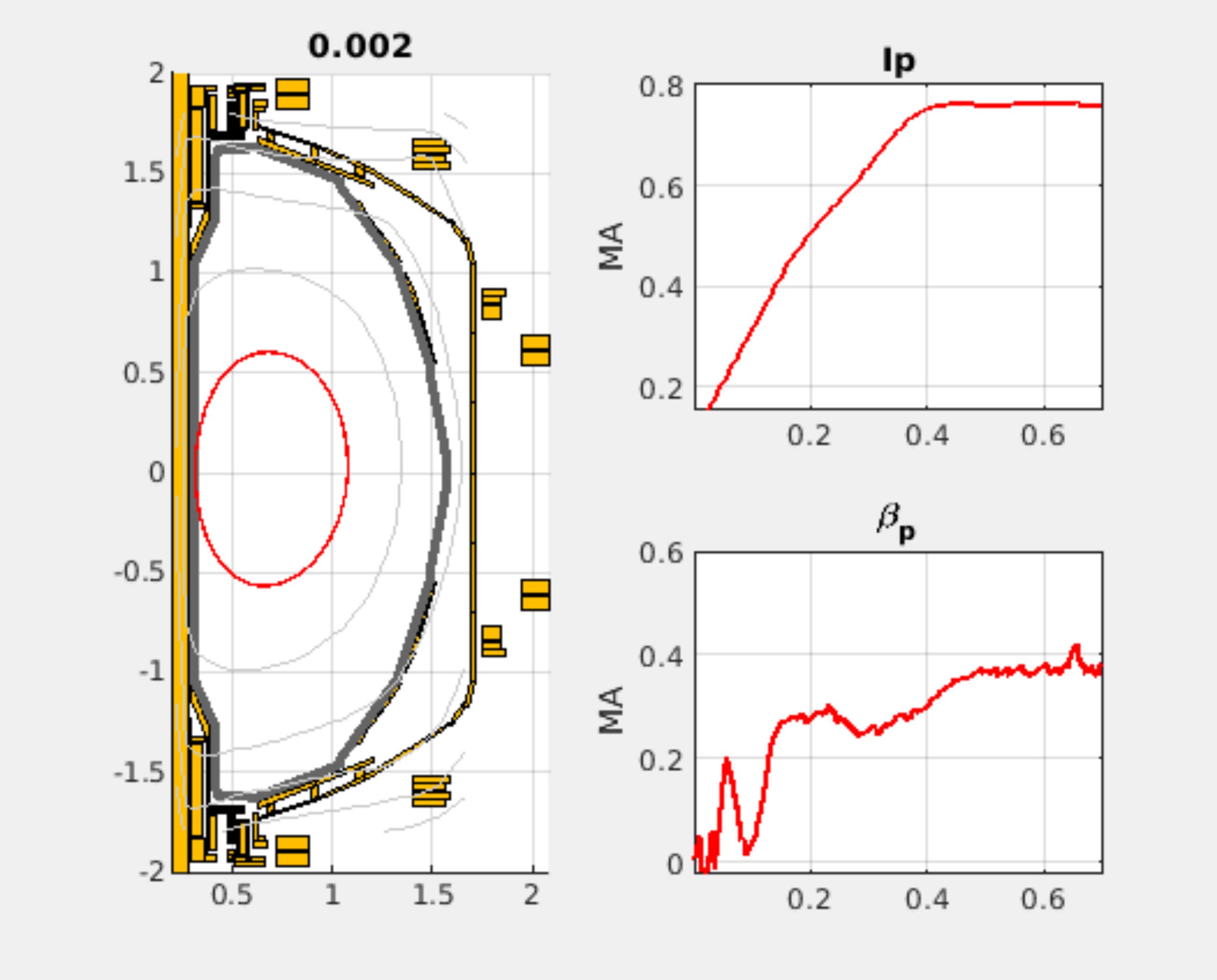

Summary

- Developed feedforward design tool and compatible shape controller
- Improve integration with vertical stability controller
- Simulation results show better control, new capabilities (e.g. divert earlier)

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